

Recall that we replaced terminal symbols appearing in the right-hand side of productions by their token names and considered the latter as non-terminals. Thus any  $zi$  that can be generated in one application can contain only one token and  $\text{INDENT}(zi)$  contains no blank space in front of this token; thus  $\text{PLOT}(t, \backslash n | \text{INDENT}(zi), 0)$  holds for some appropriate token  $t$ .

For the inductive step, note that  $zo = \text{INDENT}(zi)$  can be divided such that  $zo = zo_1 | zo_2 | \dots | zo_k$ , no  $zo_j$  contains white space suffix, and  $N_j \rightarrow^* zo_j$ . Further note that the  $zi_j$ ,  $zu_j$  and  $zo_j$  are all lexically equivalent. Since  $zu_j = \text{INDENT}(zi_j)$ ,  $zo_j$  possibly differs from  $zu_j$  only in the width of the margin of each line and by containing an extra  $\backslash n$  in the prefix of  $zo$ . The rest of the proof of this step follows from these observations and is simple but tedious requiring case analyses for each non-terminal  $N$  of the grammar. Here we present two such cases—one for the repeat statement and another for the procedure declaration.

### Case $N = \text{repeat statement}$

Clearly  $\text{TKNSEQ}(zo_1) = \text{REPEAT}$  and  $\text{TKNSEQ}(zo_{k-1}) = \text{UNTIL}$ . Also, for  $1 < j < k-1$ ,  $zo_j$  must equal  $c | zu_j$  with the margin of each line of  $zu_j$  increased by  $\text{UOI}$  blanks. Here the string  $c$  is either empty, or is  $\backslash n$  depending on the segment sequence  $\text{SEGSEQ}(zi)$ . If a segment boundary fell between  $zi_{j-1}$  and  $zi_j$ , and if  $zi_{j-1}$  did not end with a  $\backslash n$ , then  $c = \backslash n$ , else  $c = ""$ . From the definition of  $\text{SEGSEQ}$ , it follows that a segment boundary falls between  $z_{j-1}$  and  $z_j$  either because  $z_{j-1}$  terminated in a  $\backslash n$ , or in a token from  $\text{LC}$  followed by (portions of) comments, or because  $z_j$  begins a token from  $\text{LO}$ . Since  $\text{LO}$  and  $\text{LC}$  were chosen so as to make  $\text{NEWL}$  predicates true,  $\text{PLOT}(\text{repeat statement}, \backslash n | zi, 0)$  must be true.

### Case $N = \text{procedure declaration}$

We shall make further assumptions below for the sake of simplicity in this illustration. We have that  $\text{TKNSEQ}(zo_1) = \text{PROCEDURE}$ ,  $\text{TKNSEQ}(zo_2) = \text{ORDINARY}$  (the corresponding word being the name of the procedure),  $\text{TKNSEQ}(zo_3) = \text{SEMICOLON}$ , assuming that the procedure heading has no parameters, and  $\text{TKNSEQ}(zo_k) = \text{END}$ . Further assuming that the procedure has only variable declaration part, we have  $\text{TKNSEQ}(zo_4) = \text{VAR}$ . Let  $zo_5, \dots, zo_{v-1}$  correspond to this declaration such that  $\text{TKNSEQ}(zo_v) = \text{SEMICOLON}$ ,  $\text{TKNSEQ}(zo_{v+1}) = \text{BEGIN}$ . Clearly then,  $zo_{v+2}, \dots, zo_{k-1}$  correspond to the code body of the procedure. Note that the value of  $\text{NMG}()$  will be  $2 * \text{UOI}$  starting from  $zo_5$  until  $zo_v$ , both inclusive. After  $zo_{v+1}$  it becomes  $\text{UOI}$  and remains at least  $\text{UOI}$  until  $zo_k$ . As in the previous case, we see that the code body and the variable declaration and hence the procedure declaration thus meet the high level specifications.

## 7. CONCLUDING REMARKS

This section contains some remarks based on personal experience with this case study in specifying the behaviour of a medium sized program. I wrote the first version of an indenting program in late 1978 mainly as a reaction to the very long, slow and often clumsy indenting programs that were known to me at that time. A year later, I needed a class-room example of a real life program whose specification and proof are given sufficiently rigorously but with as little formalism as possible.